

Unit 7: Conics~ Learning Guide

Name: _____

Instructions:

Using a pencil, complete the following questions as you work through the related lessons. Show ALL work as is explained in the lessons. Do your best and ask your instructor if you don't understand any questions!

Lesson One

Introducing Conic Sections

1. From each of following diagrams identify the conic formed and state how each is formed by the intersection of a plane and the double-napped cone.

a.



b.



c.



2. The following are the locus definitions for different conics. List the correct conic for each locus definition.

- the absolute value of the difference of the distances from 2 given points (the 2 foci) is always constant:
- the sum of the distances from 2 given points (the 2 foci) is always constant:
- the set of points at an equal distance from a given point:
- the set of points at an equal distance from a point (the focus) and a line (the directrix):

3. A plane intersects a double-napped cone to form a circle. Assume the plane moves parallel to its original position. Describe what happens to the circle that is formed when the plane

- moves further away from the vertex
- is at the vertex of the double-napped cone.

c) moves closer to the vertex

Lesson Two

Graphing Circles and Rectangular Hyperbolas

1. Graph the following conics on the same grid.

A. $x^2 + y^2 = 1$

B. $x^2 + y^2 = 9$

C. $x^2 + y^2 = 36$

2. State the center and the radius for each of the following circles.

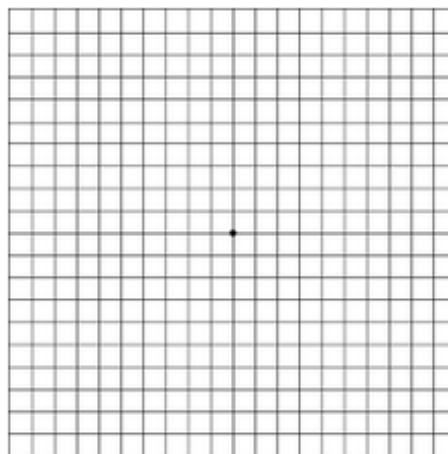
a. $x^2 + y^2 = 4$

b. $x^2 + y^2 = 11$

c. $x^2 + y^2 = \frac{16}{25}$

3. Write the equation of a circle with a center at (0,0) and a radius of $\sqrt{20}$.

4. Write the equation of a circle with a center at (0,0) and a radius of $\frac{9}{2}$.



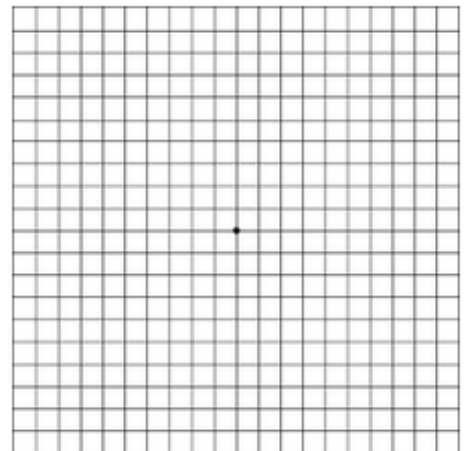
5. Write the equation of a circle with a center at (0,0) and a diameter that is 8 units long.

6. Write the equation of a circle where the endpoints of the diameter are at (6,7) and (-6, -7).

7. Write the equation of a hyperbola that has a center at (0,0) vertices at (1,0) and (-1,0) and the equation of one asymptote is $y = -3x$.

8. Prove that the point (2,3) is not on the graph of the hyperbola $4x^2 - 9y^2 = -36$

9. Graph the hyperbola $x^2 - y^2 = -36$. Show the asymptotes on your graph.

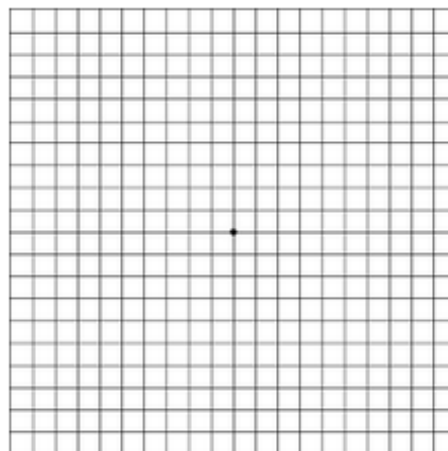


Lesson Three

Expanding and Compressing Graphs of Conic Sections

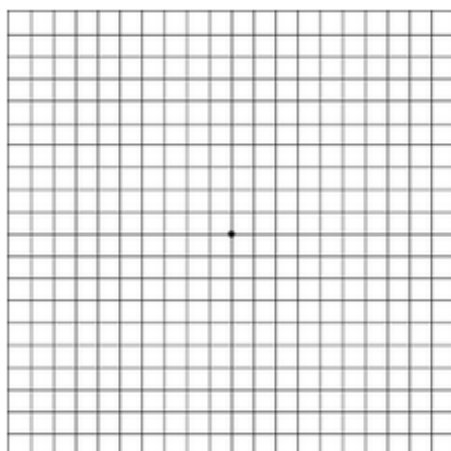
1. Explain how an ellipse is different from a circle in terms of transformations.
2. Convert the following ellipse to standard form: $4x^2 + 9y^2 = 36$
3. State the length of the major axis and the minor axis, and the coordinates of the vertices for the following ellipse. State if the major axis is vertical or horizontal: $25x^2 + 16y^2 = 400$

4. Sketch the graph for the following ellipse: $16x^2 + y^2 = 64$

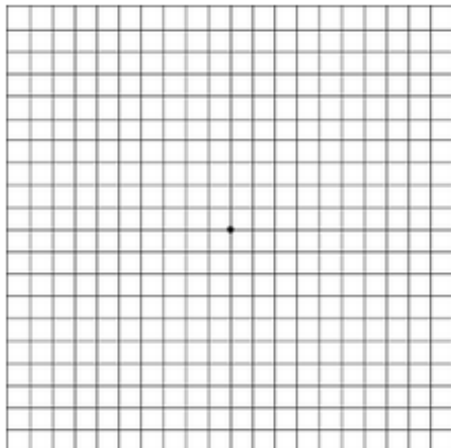


5. Sketch the graph of each of the following parabolas. Label the coordinates of the vertex, and the coordinates of two other points.

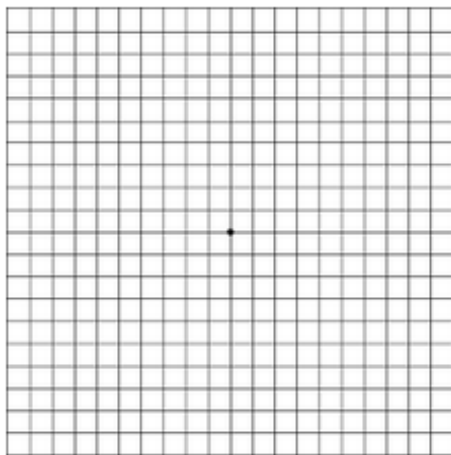
a. $y = -\frac{1}{2}x^2 - 3$



b. $y = 4x^2 + 1$



c. $x = -2y^2 - 3$



6. Describe the direction of opening and the equation of the axis of symmetry for the following parabolas.

a. $y = 2x^2$

b. $x = -7y^2$

c. $y = -x^2 - 3$

d. $x = \frac{4}{5}y^2 + 2$

7. A parabola is symmetric about the x-axis and passes through the point (5, 2). What must be another point on the parabola?

Lesson Four

Translating Graphs of Conic Sections

1. State the coordinates of the center for each of the following conics listed below.

a. $(x+4)^2 + y^2 = 36$

b. $4(x-7)^2 + 16(y+4)^2 = 64$

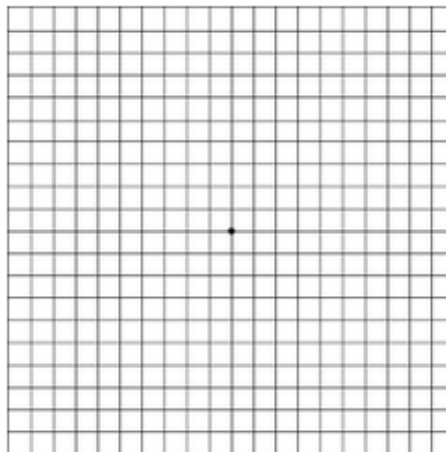
2. State the coordinates of the vertices for each of the following conics.

a. $(x+1)^2 - 4(y-3)^2 = -36$

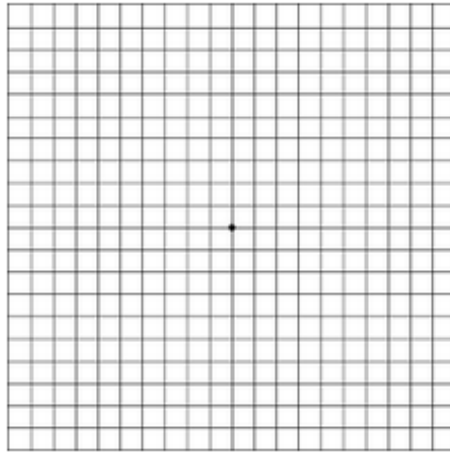
b. $\left(x + \frac{3}{2}\right)^2 - 9(y+1)^2 = 49$

3. Sketch the graph each for each of the following conics.

a. $(x-1)^2 + (y+3)^2 = 20$



b. $25(x-2)^2 - 9(y+4)^2 = -225$



4. Write the equations for each of the following conics.

a. Parabola with vertex at $(-3, 2)$, horizontal axis of symmetry, and passing through the point $(-5, 4)$.

b. Hyperbola that has a horizontal transverse axis 6 units long, and a conjugate axis 20 units long. Hyperbola has been translated 1 unit up and 4 units right from $(0,0)$.

Lesson Five

The Equation of a Conic Section in General Form

1. Change each of the following equations into general form.

a. $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{3} = 1$

b. $\frac{(x-3)^2}{2} - \frac{(y+3)^2}{5} = -1$

2. Which conic could be represented by each equation. Explain your reasoning.

a. $4x^2 + 6y^2 + 3x - 12y - 24 = 0$

b. $2y^2 - 4x - 2y + 5 = 0$

3. Which conic could be represented by the equation, $Ax^2 + Cy^2 + Dx + Ey = 0$ given the following conditions:

a. $A = C \neq 0$

b. $AC > 0$ and $A \neq C$

4. Determine the restrictions on the constants (parameters) A, C, D, and E in the equation $Ax^2 + Cy^2 + Dx + Ey = 0$ given the following conditions on each conic listed:

a. A parabola with a vertical axis of symmetry with a vertex at (0, 0)

b. A circle with a center at a point not on the x or y axis.

5a. Given the conic $\frac{x^2}{4} + \frac{y^2}{9} = 1$, write the new equation in standard form after a horizontal expansion by a factor of 4 and a vertical compression by a factor of 1/3.

b. Given the conic $\frac{x^2}{25} - \frac{y^2}{16} = 1$, write the new equation in standard form after a horizontal compression by a factor of 1/2 and a vertical compression by a factor of 1/8.

Lesson Six

The Equation of a Conic Section in Standard Form

1. Convert from general to standard form then answer the following questions:

a. $2x^2 + 3y^2 - 12x - 12y - 10 = 0$ Find the length of the major and minor axes

b. $4x^2 + 4y^2 - 12x - 30 = 0$ Find the center and length of diameter

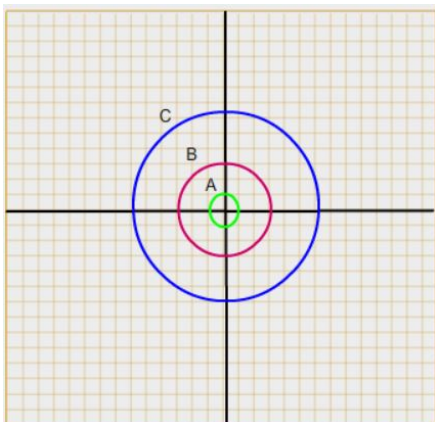
c. $36x^2 + 64y^2 + 108x - 128y - 431 = 0$ Find the vertices

ANSWERS***Lesson One Introducing Conic Sections***

1. a) Circle – when a plane intersects a double-napped cone such that the plane is parallel to the axis or perpendicular to the axis.
 b) Parabola – when a plane intersects a double-napped cone such that the plane is parallel to the generator.
 c) Ellipse – when a plane intersects a double-napped cone such that the plane is neither perpendicular nor parallel to the axis and the angle of intersection is greater than the generator angle.
- 2a) Hyperbola b) Ellipse c) Circle d) Parabola 3. a) The radius of the circle increases so the circle is larger b) The radius of the circle becomes infinitely small so a point is formed
 c) The radius of the circle decreases so the circle is smaller

Lesson Two Graphing Circles and Rectangular Hyperbolas

1.



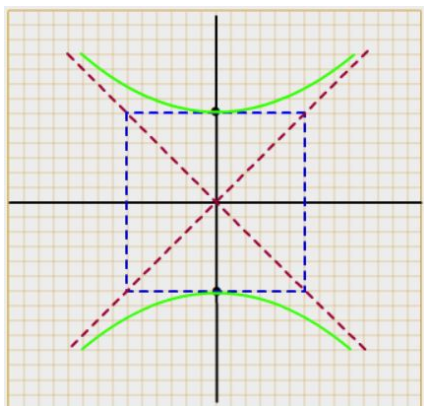
2a. $(0,0)$ 2 b. $(0,0)$ $\sqrt{11}$ c. $(0,0)$ $\frac{4}{5}$

3. $x^2 + y^2 = 20$ 4. $x^2 + y^2 = \frac{81}{4}$ 5. $x^2 + y^2 = 16$

6. $x^2 + y^2 = 85$ 7. $x^2 - \frac{y^2}{9} = 1$

8. $4(2)^2 - 9(3)^2 = -36$
 $16 - 81 \neq -36$

9.

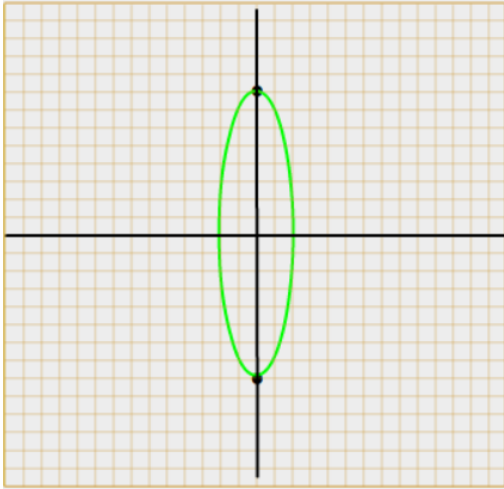


Lesson Three Expanding and Compressing Graphs of Conic Sections

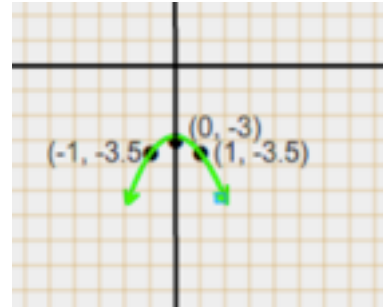
1. An ellipse is a circle that has either been horizontally and vertically compressed or expanded.

2. $\frac{x^2}{9} + \frac{y^2}{4} = 1$ 3. Major axis: 10 Minor axis: 8 Vertices: (0,5) and (0,-5), Major axis vertical

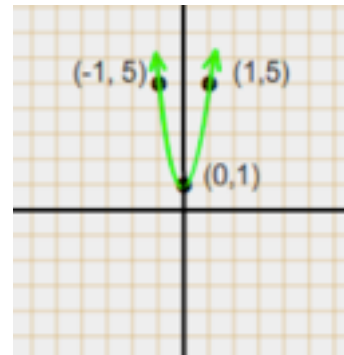
4.



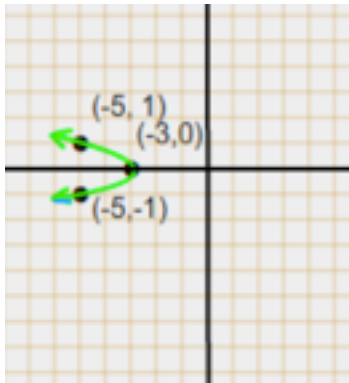
5. a)



b)



5. c)



6. a) Direction of opening: up

Equations of axis of symmetry: $x = 0$

b) Direction of opening: left

Equations of axis of symmetry: $y = 0$

c) Direction of opening: down

Equations of axis of symmetry: $x = 0$

d) Direction of opening: right

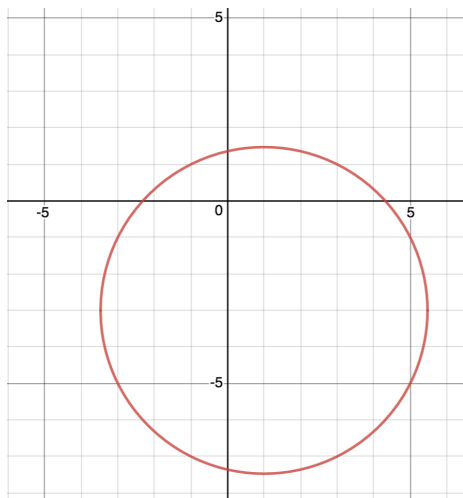
Equations of axis of symmetry: $y = 0$

7. (5, -2)

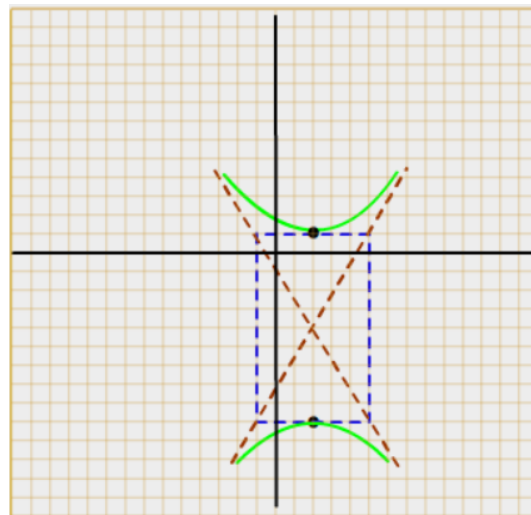
Lesson Four Translating Graphs of Conic Sections

1a) $(-4,0)$ b) $(7,-4)$ 2a) $(-1,6)$ and $(-1,0)$ b) $\left(\frac{11}{2}, -1\right)$ and $\left(-\frac{17}{2}, -1\right)$

3. a)



b)



4a) $x+3 = -\frac{1}{2}(y-2)^2$ b) $\frac{(x-4)^2}{9} - \frac{(y-1)^2}{100} = 1$

Lesson Five - The Equation of a Conic Section in General Form

1a) $3x^2 + 4y^2 - 6x + 16y + 7 = 0$ b) $5x^2 - 2y^2 - 30x - 12y + 37 = 0$

2.a) Ellipse as $A \neq C$ and $AC > 0$ b) Parabola as $A=0$ and $C \neq 0$ 3a) Circle b) Ellipse

4a) $A \neq 0, C=0, D=0, E \neq 0$ b) $A=C, D \neq 0, E \neq 0$ 5a) $\frac{x^2}{64} + y^2 = 1$ b) $\frac{4x^2}{25} - 4y^2 = 1$

Lesson Six - The Equation of a Conic Section in Standard Form

$$\frac{(x-3)^2}{20} + \frac{(y-2)^2}{\frac{40}{3}} = 1 \rightarrow \text{Major axis: } 4\sqrt{5} \text{ Minor axis: } \frac{4\sqrt{30}}{3}$$

1a)

b) $\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{39}{4} \rightarrow \text{Center: } \left(\frac{3}{2}, 0\right) \text{ Diameter: } \sqrt{39}$

c) $\frac{\left(x + \frac{3}{2}\right)^2}{16} + \frac{(y-1)^2}{9} = 1 \rightarrow \text{Vertices: } \left(-\frac{11}{2}, 1\right) \text{ and } \left(\frac{5}{2}, 1\right)$